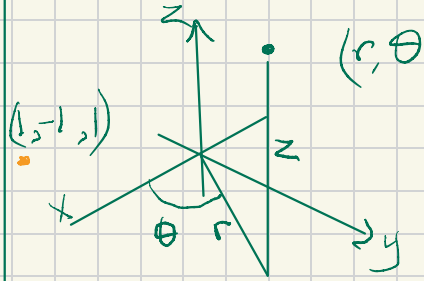
Cylindrical and Spherical coordinates

We learn

- polar coordinates in 2-D (review) (r, θ)
- cylindrical coordinates in 3-D (r, θ, z)
- Spherical coordinates in 3-D.

Cylindrical coordinates



(r, θ, z) is $(r\cos\theta, r\sin\theta, z)$ in Cartesian coordinates.

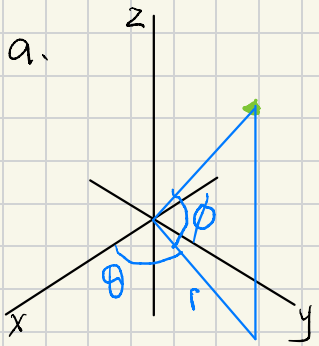
Example:
Find the cylindrical coordinates of (x,y,z) = (1,-1,1)

Soln $\left(\sqrt{2}, \frac{7\pi}{4}, 1\right)$

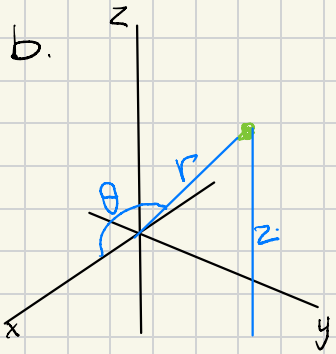
Pre-class Warm-up!!!!

Which picture best describes cylindrical coordinates for R^3 ?

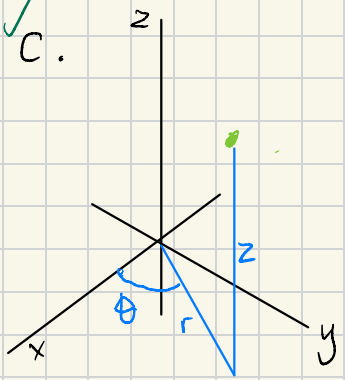
a.



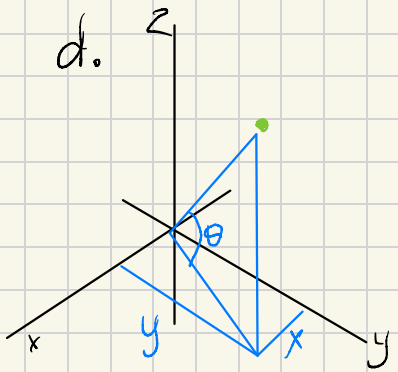
b.



✓ c.



d.

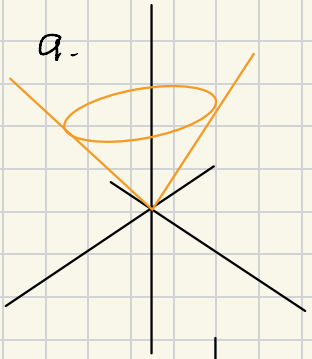


What are the surfaces in cylindrical coordinates (r, θ, z).

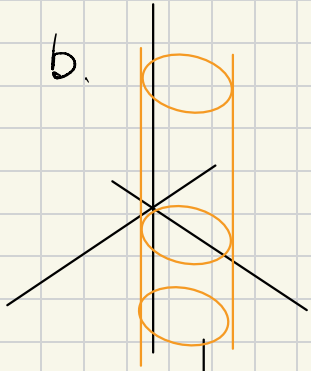
- $z^2 = r^2$ e.
- $z = r^2$ c.
- $r = 4 \sin\theta$ b.

What is the region
r in [0,1], theta in [0,π], z in [-1,1] ? f.

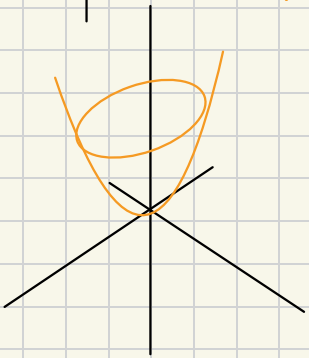
a.



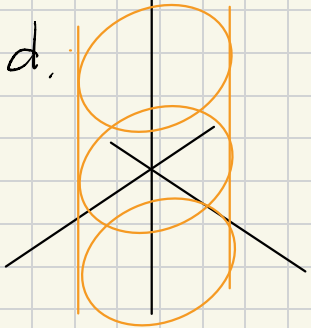
b.



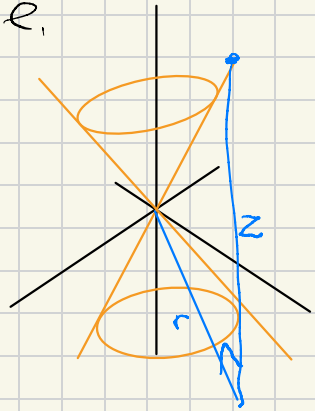
c.



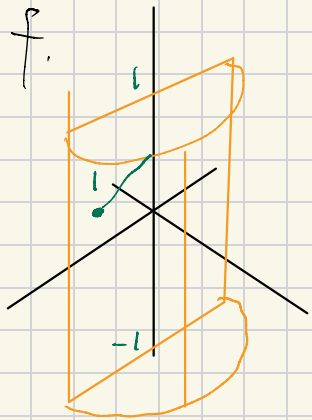
d.

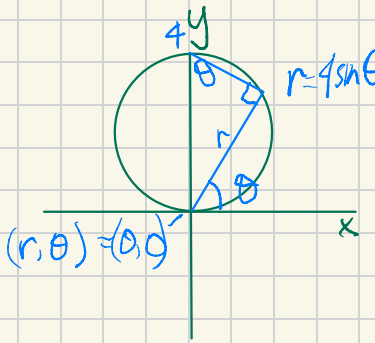


e.

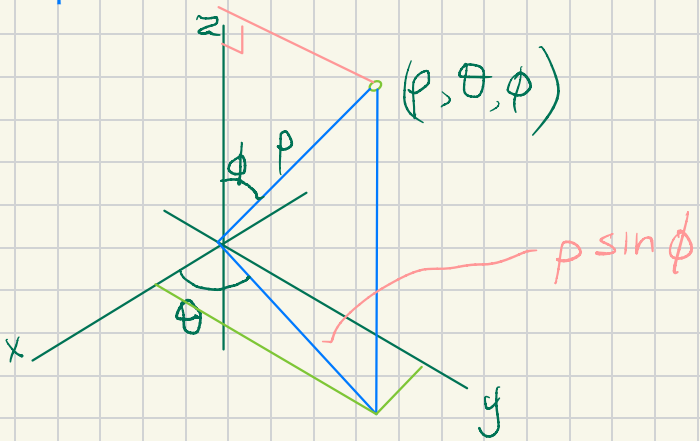


f.





Spherical coordinates



Usually $\rho \geq 0$, $0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq \pi$.

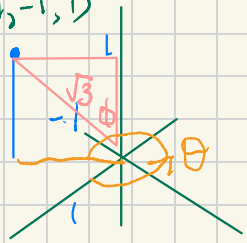
We have

$z = \rho\cos\phi$

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

Cartesian $(1,-1,1)$



$\rho = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\phi = \frac{7\pi}{4}$

$\phi = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Examples:

- Put (x,y,z) = (1,-1,1) in spherical polar coordinates
- Sketch the surface p = 2 $\rho = 2$ sphere of radius 2, center origin
- Sketch surface phi = π / 4
- Sketch the surface rho sin phi = 2 distance to z-axis is 2